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The Neutral Rate of Interest in Canada

by Rhys R. Mendes

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Abstract

A measure of the neutral policy interest rate can be used to gauge the stance of monetary policy. We define the neutral rate as the real policy rate consistent with output at its potential level and inflation equal to target after the effects of all cyclical shocks have dissipated. This is a medium- to longer-run concept of the neutral rate. Under this definition, the neutral rate in Canada is determined by the longer-run forces that influence savings and investment in both the Canadian and global economies. Structural forces have likely reduced the neutral rate by more than a percentage point since the mid-2000s. The Bank's estimates of the real neutral policy rate currently stand in the 1 to 2 per cent range, or 3 to 4 per cent in nominal terms. The current gap between the policy rate and the neutral rate reflects policy stimulus in response to significant excess supply and in the face of continuing headwinds. As long as these headwinds persist, a policy rate below neutral will be required to maintain inflation sustainably at target.

JEL classification: E40, E42, E43, E50, E52, E58
Bank classification: Interest rates; Transmission of monetary policy

Résumé

Il est possible d'évaluer l'orientation de la politique monétaire en utilisant une estimation du taux directeur neutre. Le taux neutre désigne le taux directeur réel compatible avec la production lorsque celle-ci se trouve à son niveau potentiel et que l'inflation a atteint la cible, une fois passés les effets de tous les chocs cycliques. Il s'agit d'une conception à moyen et long terme du taux neutre. Sous cette forme, le taux neutre au Canada est déterminé par des forces à long terme qui influencent l'épargne et l'investissement dans l'économie canadienne comme à l'échelle internationale. Ces facteurs structurels ont fait vraisemblablement baisser le taux neutre de plus d'un point de pourcentage depuis le milieu des années 2000. La Banque estime que la fourchette actuelle du taux directeur neutre va de 1 % à 2 % en valeur réelle (ou de 3 % à 4 % en valeur nominale). L'écart actuel entre le taux directeur et le taux neutre traduit l'incidence de la détente monétaire imprimée en réaction à une offre excédentaire substantielle et face à des vents contraires persistants. Tant que dureront ces vents, le taux directeur devra être établi au-dessous du taux neutre afin que l'inflation reste au niveau cible.

Classification JEL : E40, E42, E43, E50, E52, E58
Classification de la Banque : Taux d'intérêt; Transmission de la politique monétaire

There is a certain rate of interest on loans which is neutral in respect to ... prices, and tends neither to raise nor to lower them. (Knut Wicksell, *Interest and Prices*, 1898)

1 Introduction

A measure of the neutral stance of monetary policy provides a benchmark against which one can gauge the degree of monetary stimulus in an economy. The Bank of Canada implements monetary policy through adjustments to its policy interest rate. Thus, an estimate of the neutral policy rate is an essential input into the models and analysis that inform the monetary policy decision-making process.

The neutral or natural rate of interest does not have a unique definition. Lack of clarity about the precise concept being used can be a source of confusion in discussions about the neutral rate. **Box 1** provides an overview of the most commonly used concepts. We define the neutral rate as the real policy rate consistent with output at its potential level and inflation equal to the 2 per cent target *after the effects of all cyclical shocks have dissipated*. This is a medium- to long-run concept which varies over time with lower-frequency factors such as demographic change and shifts in trend productivity growth. The deviation of the actual policy rate from neutral – the interest rate gap – is a measure of the stance of monetary policy.

The Bank's estimates of this concept of the neutral policy rate currently stand in the range of 1 to 2 per cent in real terms, or 3 to 4 per cent in nominal terms. This range reflects the range of estimates from the various approaches discussed in this paper. Thus, with the nominal policy rate currently at 1 per cent, monetary policy is stimulative in Canada.[1] As Wilkins (2014) explains, this stimulus is needed to close the output gap and maintain inflation sustainably at target in the face of persistent headwinds. However, this does not imply that the policy rate will necessarily return to neutral as soon as the output gap is closed and inflation is at target. In particular, a non-neutral policy stance may be required to offset the effects of headwinds or tailwinds and keep inflation at target. Thus, the policy rate may return to neutral *after* inflation reaches target and the output gap is closed.[2]

The Bank's estimates of the neutral rate have declined by more than 1 percentage point since the mid-2000s as a result of structural changes in the Canadian and global economies.[3] This decline largely reflects a lower global neutral rate and reduced potential output growth in Canada. The remainder of this paper lays out a framework for analyzing the determinants of the neutral rate, and documents how the evolution of these determinants has put downward pressure on the neutral rate in recent years. Section 2 explains the link between the neutral rate and the determinants of savings and investment.

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[1] Staff use the midpoint of the range as an anchor in our macroeconomic models.

[2] See Technical Box 2 in the July 2011 *Monetary Policy Report* for a more complete explanation of this point.

[3] In the original version of ToTEM, the Bank's main macroeconomic model of the Canadian economy, the steady-state real interest rate was set at 3.25. See the discussion on page 57 of Murchison and Rennison (2006).

Section 3 documents the evolution of these determinants in Canada and abroad. Section 4 provides estimates of the neutral rate in Canada. Section 5 offers some conclusions.

Box 1

Alternative definitions of the neutral rate

The common use of several different concepts of the neutral rate is sometimes a source of confusion. At any given time, there can be material quantitative disparities between different measures of the neutral rate. Definitions differ mainly with respect to the horizon over which they are consistent with a zero output gap. There are at least two commonly used horizons:

- **Contemporaneous**: This is the rate of interest that would ensure a zero output gap in every period. This concept was popularized by Woodford (2003). It is more useful as an indicator of the policy rate that is warranted by current economic conditions, rather than as a benchmark against which to gauge the degree of policy stimulus.
- **Medium- to long-run**: This is the rate of interest that should prevail after the effects of business cycle shocks have dissipated. This concept of the neutral rate would be influenced by medium- to longer-term forces such as demographic change. Deviations of the actual policy rate from this measure of neutral gauge the stance of monetary policy.

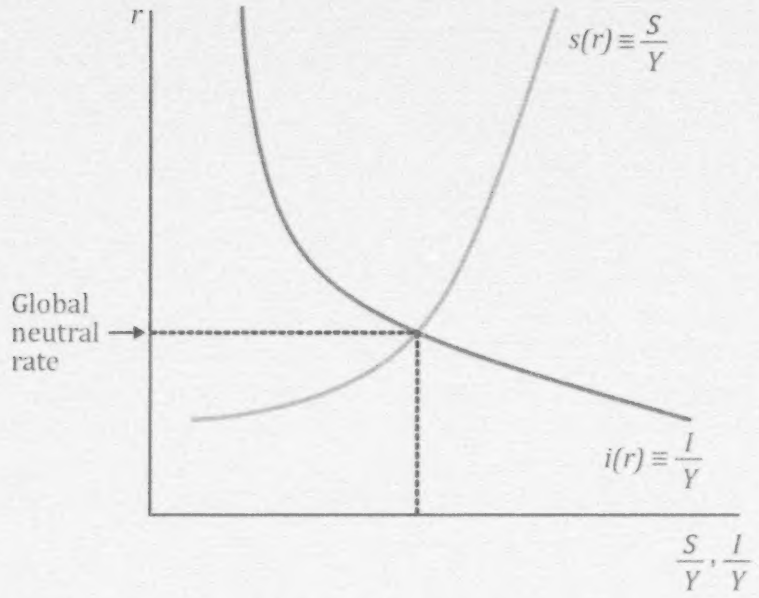
At the current juncture, cyclical headwinds would cause the contemporaneous measure to be well below the medium- to long-run measure. As a longer-run anchor for the policy rate, the medium- to long-run concept of neutral is most relevant.

2 Savings, Investment and the Neutral Rate

2.1 The global neutral rate

All investment must ultimately be financed by savings. At a global level (or in a closed economy), this means that investment and savings must always be equal. But the equality of savings and investment is not automatic. Households choose how much to save based on their current and expected future income levels and the rate of return they can earn on their savings (the interest rate). Similarly, firms choose how much to invest in new capital based on the cost of financing (the interest rate) and the expected marginal return on capital investment (the marginal product of capital).

Figure 1: Long-run global savings and investment



Note: The blue line denotes the global savings rate ($s = S/Y$) as a function of the global real interest rate (r) and the red line denotes the global investment-output ratio ($i = I/Y$) as a function of r. Both curves represent long-run relationships (after the cyclical effects of shocks have dissipated).

In the short run, adjustments in the level of income (or output) can make desired savings equal to desired investment at any given interest rate. However, in the long run, we would expect output to be at its potential (or full employment) level. Thus, after the cyclical effects of all shocks have dissipated, only the interest rate can equilibrate savings and investment. At a global level, the interest rate that ensures saving is equal to investment in the long run is the global neutral rate (Figure 1). This provides a framework for thinking about the determinants of the global neutral rate: we can analyze the determinants of saving and investment. But in order to analyze the neutral rate in Canada, we need a framework for taking account of global and domestic influences.

2.2 Global and domestic drivers of the Canadian neutral rate

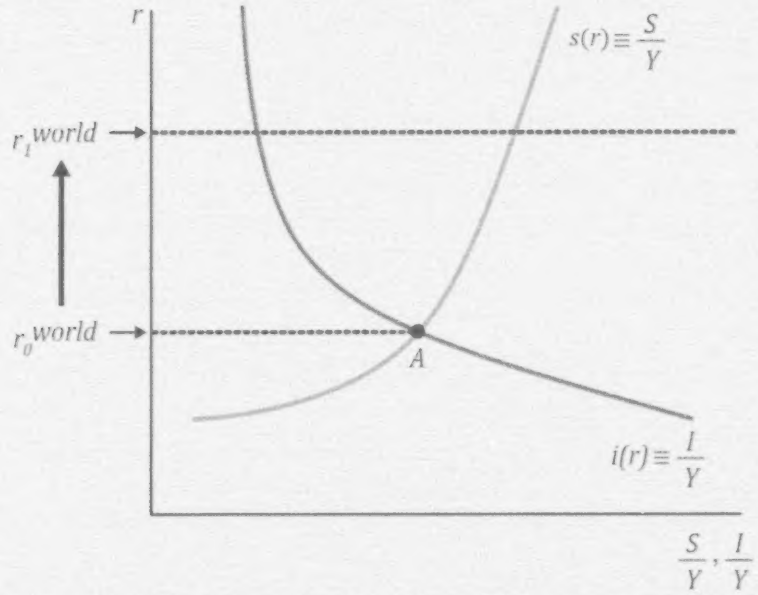
In an open economy like Canada, savings need not be equal to investment. Canada currently invests more than it saves. The difference, the external current account deficit, is financed by inflows of foreign capital. This implies that financing conditions for domestic investment depend not only on the availability of domestic savings, but also on the net supply of foreign savings (i.e., the supply of foreign savings net of that absorbed by investment in the rest of the world). In order to analyze the implications of open-economy considerations, we will examine two cases: (i) Canada as a pure price-taker, and (ii) an endogenous risk premium.

Case 1: The Mundellian case: Canada as a pure price-taker

We begin with a case in which we abstract from risk, so that international capital flows fully equalize Canadian and international interest rates in steady state. Moreover, we assume that Canada is small relative to the rest of the world, so domestic Canadian economic developments do not influence the global real interest rate. This is essentially the case studied by Mundell (1963).

For illustrative purposes, suppose that domestic saving and investment are initially equal at the world real interest rate (Point A in Figure 2), and then there is a shock that permanently raises the global interest rate to r_1^{world}. This will induce Canadians to invest abroad, since desired saving by domestic residents at the new world real interest rate exceeds domestic investment (Figure 2). These outflows of capital will drive down the value of the Canadian dollar and create inflationary pressures that ultimately cause domestic interest rates to rise until they are equal to the global interest rate. The higher interest rates will reduce the optimal capital intensity of production, thus lowering investment demand. The shortfall in demand created by the decline in investment will be offset by stronger net exports resulting from the depreciation of the currency.

Figure 2: Savings and investment at the world real interest rate



Note: The blue line denotes the Canadian savings rate ($s = S/Y$) as a function of the real interest rate (r) and the red line denotes the Canadian investment-output ratio ($i = I/Y$) as a function of r. Both curves represent long-run relationships (after the cyclical effects of shocks have dissipated).

In this environment, domestic factors are irrelevant for the determination of the real rate of interest that will prevail in Canada in the long run. In other words, the neutral rate in Canada would be determined solely by global factors. Nevertheless, knowledge of the domestic factors that determine

savings and investment is important for understanding the long-run value of the exchange rate, the external current account, and the composition of demand. With higher interest rates and a weaker exchange rate, Canada would run a current account surplus and Canadian GDP would consist of more exports and less investment (relative to the case with a lower global interest rate).

Case 2: An endogenous risk premium

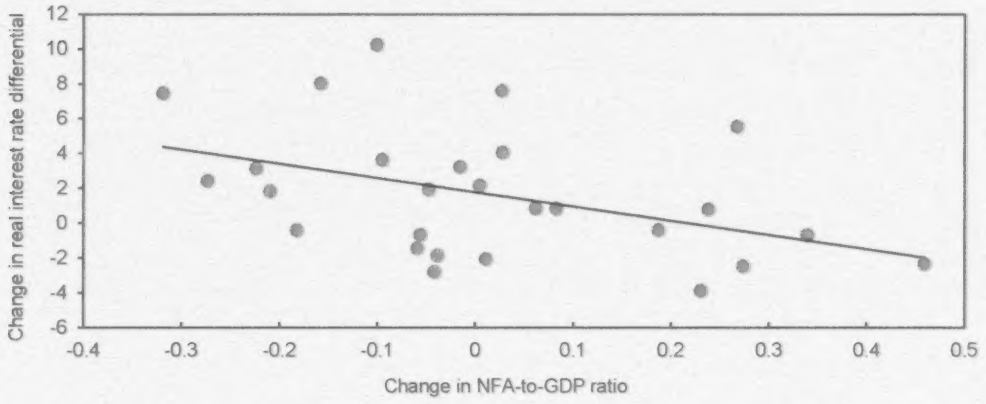
Alternatively, Canadian interest rates (r) may differ from global rates (r^{world}) in the long run by an endogenous risk premium (ϕ):

$$r = r^{world} + \phi.$$

This risk premium is assumed to depend on Canada's net foreign asset position: the risk premium decreases as Canadians accumulate foreign assets. In this case, domestic Canadian factors would influence the long-run interest rate differential, but only to the extent that they affect the risk premium.

As in the previous case, suppose that domestic saving and investment are initially equal at the global interest rate, and then a shock raises the global rate. As before, this will induce Canadians to save more and Canadian businesses to undertake less investment spending, resulting in a current account surplus. The difference with respect to the Mundellian case is that, as Canadians accumulate foreign assets, foreigners will perceive lending to Canadians as less risky. Thus, the risk premium, ϕ, will shrink, ultimately turning negative and allowing Canadian interest rates to be lower than foreign interest rates. **So, with an endogenous risk premium, it is possible, in principle, for Canada's real interest rate to be influenced by domestic factors, even in the long run.**

Figure 3: **Change in real interest rate differentials versus change in NFA-to-GDP ratio**



Notes: The NFA-to-GDP ratio is the 10-year average of the net foreign asset position of a country relative to the 10-year average of GDP of the same country. The change is the difference between two 10-year periods. Interest rates used are 10-year averages of the 3-month treasury bill rate. Money market rates are used where treasury bill rates were not available. The interest rate differentials are relative to the United States. The countries included are Australia, Canada, Denmark, Germany, Great Britain, Hungary, Israel, Japan, Mexico, New Zealand, Norway, South Korea, Sweden, Switzerland and Turkey.
Sources: Haver, Bloomberg and the External Wealth of Nations Mark II database

The existence of country-specific risk premia that depend on net foreign asset (NFA) positions has some empirical support. Figure 3 shows that improvements in a country's NFA-to-GDP ratio tend to be associated with declines in their interest rate differential versus the United States. That is, countries with stronger NFA positions tend to have lower risk premia. The figure is constructed using decadal averages of interest rate differentials and NFA-to-GDP ratios in order to focus on lower-frequency developments. More formal empirical analyses confirm the story that emerges from Figure 3 (see Lane and Milesi-Ferretti 2001; Rose 2010). Thus, our analysis allows for the possibility that the determinants of domestic savings and investment affect the neutral rate through their influence on NFA and the risk premium.

3 The Evolution of Determinants of the Neutral Rate

This previous section sketched a framework for analyzing the determinants of the neutral rate: we can analyze the determinants of saving and investment in Canada and abroad. The key domestic determinants of the neutral rate are summarized in Table 1. Given our medium- to longer-run definition of neutral, we focus on variables that have shifted or may have shifted for structural rather than cyclical reasons. Importantly, we use the 1999–2006 average as an illustrative pre-crisis benchmark.

Table 1: Key domestic determinants of the neutral rate in Canada

	1999-2006 average	2013 value	2014-16 average	Impact on neutral rate
Potential output growth (g)	2.8%	1.9%	1.9%	↓
- *Trend labour input growth*	*1.4%*	*0.8%*	*0.6%*	↓
- *Trend labour productivity growth*	*1.4%*	*1.1%*	*1.3%*	↔
Household effective credit spread	210 bps	230 bps		↓
Firm effective credit spread	145 bps	190 bps		↓

Note: bps = basis points

3.1 Lower potential growth reduces investment demand

In order to identify the determinants of investment demand, it is helpful to start by asking how much investment would be needed to keep the capital-output ratio constant. In the absence of investment, two natural forces would tend to shrink the capital-output ratio over time:

- **Depreciation**: The physical stock of capital wears out over time and must be discarded. If it is not replaced, the capital-output ratio declines.
- **Output growth**: For a given capital stock, output growth reduces the capital intensity of production (i.e., it raises the denominator in the capital-output ratio).

Thus, in order to keep the capital-output ratio constant, the share of output devoted to investment must be increasing in the rate of output growth (g) and the rate of depreciation (δ). One can show that the investment-output ratio (I/Y) required to maintain a constant capital-output ratio is[4]

$$\frac{I}{Y} = \kappa \times (g + \delta), \tag{1}$$

where $\kappa = K/Y$ is the capital-output ratio.

As a consequence, lower output growth implies lower investment demand as a share of output. This is the familiar accelerator effect. It suggests that slowing potential output growth should reduce the investment-output ratio by κ times the change in potential growth.

But the assumption of a constant capital-output ratio is likely too restrictive. Economic theory suggests that the intensity with which capital is used in the production of output should depend upon the user cost of capital. Among other things, a higher real interest rate raises the cost of financing capital expenditures and should therefore induce firms to substitute away from capital toward other factors of production (such as labour). As a consequence, the capital-output ratio should be a decreasing function of the real interest rate, r.[5] Using this yields an inverse relationship between investment and the interest rate:

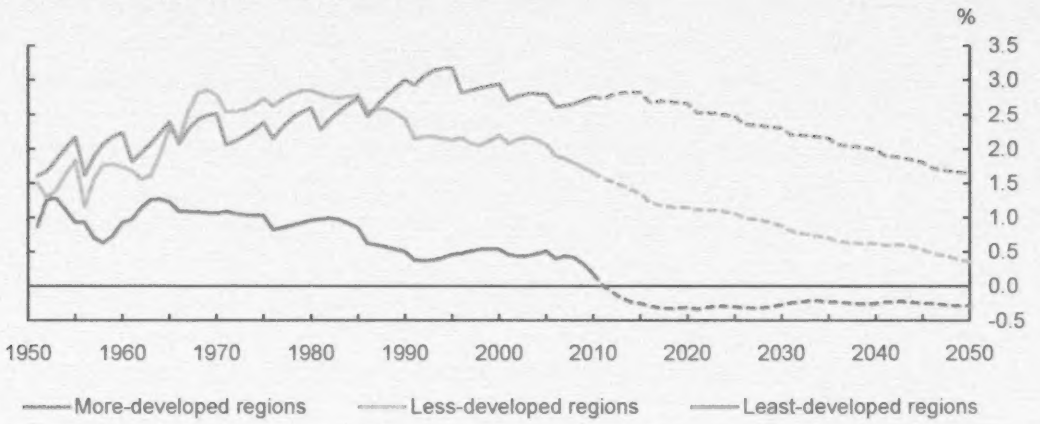
$$\frac{I}{Y} = \kappa(r) \times (g + \delta). \tag{2}$$

In the long run, output growth depends on the growth rate of labour input and labour productivity. Globally, the growth rate of the working-age population is on a downward trend (Figure 4). In major advanced economies, productivity growth has also declined in recent years (Figure 5).

[4] This relationship can be derived by taking the capital accumulation equation ($K_{t+1} = (1 - \delta)K_t + I_t$) and manipulating it under the steady-state assumption.

[5] The capital-output ratio will also be a decreasing function of the depreciation rate. With a Cobb-Douglas production function, changes in the capital-output ratio completely offset the direct impact of changes in the depreciation rate on the investment-output ratio if $r = g$. More generally, if r and g are not too different, then changes in the depreciation rate will have a minimal impact on the investment-output ratio. Partly for this reason, we do not focus on changes in the depreciation rate.

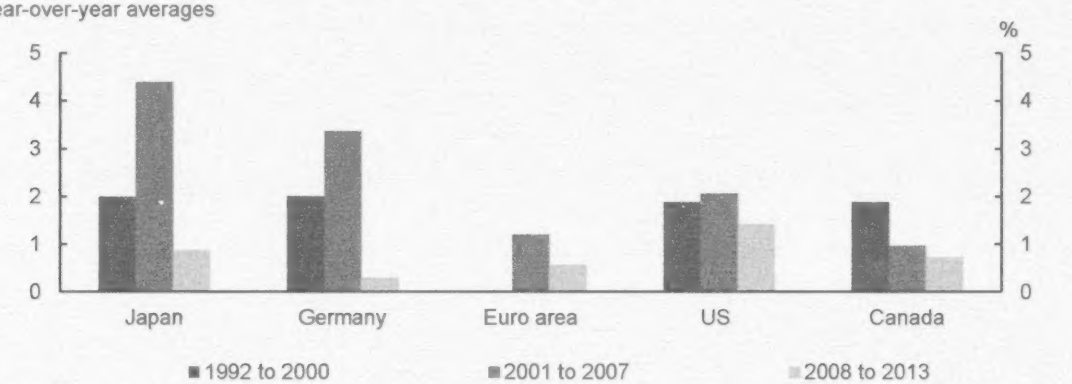
Figure 4: **Working age population growth**



Note: Working age population includes individuals aged 15-64
Sources: United Nations, Haver Analytics

Last observation: Historical data - 2010
Projected data - 2050

Figure 5: **Growth rate averages of GDP per hour worked**
Year-over-year averages



Source: OECD

Last observation: 2013

In Canada, the Bank expects potential growth over the 2014–16 period to be 0.9 percentage points below its pre-crisis (1999–2006) average. Estimates of the total net capital-output ratio have ranged from 1 to 2 in recent decades. This means that, with a constant real interest rate (and therefore a constant capital-output ratio), the investment-output ratio would have to decline by between 1 and 2 percentage points for every one percentage point decline in the growth rate. This is the amount by which the investment demand schedule would shift to the left as the growth rate declines. Taking into account expected potential growth, the estimated impact on the investment-output ratio would be 0.9

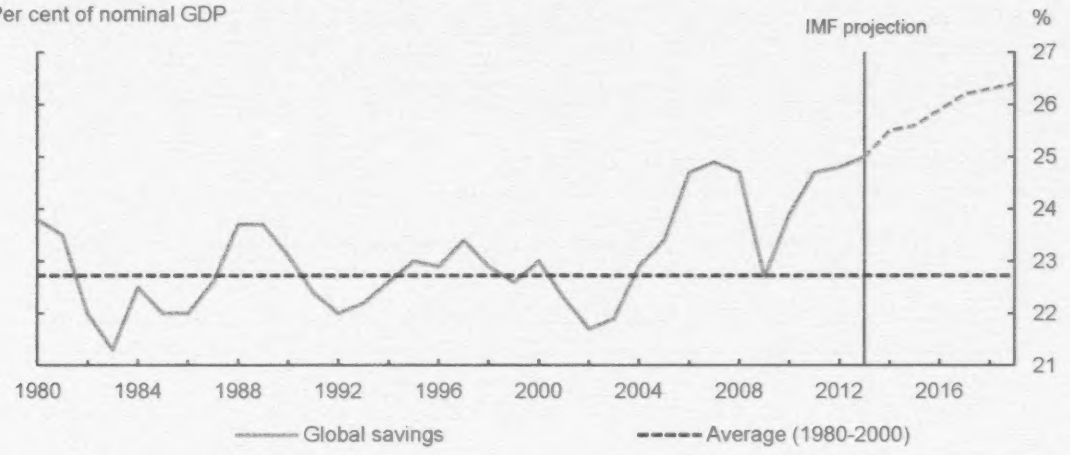
to 1.8 percentage points at a given interest rate. In isolation, this effect would suggest that lower potential growth should lead to a lower neutral rate.[6]

3.2 Global savings rate remains elevated

The global savings rate began to increase in the early 2000s. This rise was interrupted by the crisis, but the International Monetary Fund (IMF) expects it to continue in the future (Figure 6). The IMF projects the global savings rate to average 26 per cent over the 2014–19 period, more than 3 percentage points above its 1999–2006 average.

Figure 6: **The savings rate is expected to continue rising**

Per cent of nominal GDP



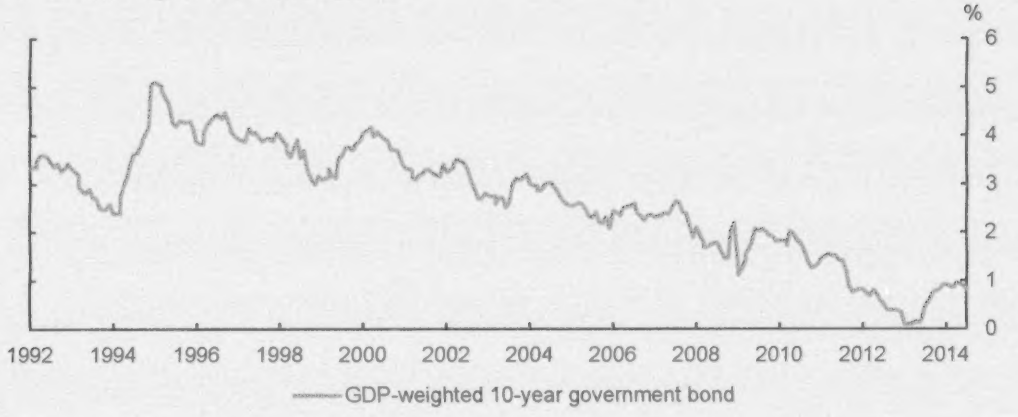
Source: IMF, *World Economic Outlook*, April 2014

Last observation: Historical data - 2013
Projected data - 2019

In isolation, an elevated or rising savings rate is not necessarily indicative of a shift in the global savings schedule (Figure 1). In equilibrium savings must equal investment, so a rising savings rate will necessarily be accompanied by a rising investment rate. However, rising savings since the early 2000s coincided with declining global real interest rates (Figure 7). Thus, the global supply of savings was likely increasing at a faster pace than the global demand for investment.

[6] In equilibrium, a lower interest rate would lead to an increase in the capital-output ratio that would partially offset the impact of the growth shock on investment demand. This is the oft-cited capital deepening that many expect to be associated with lower labour force growth. However, given that capital deepening is the *result* of a lower interest rate, it will not completely offset the constant-interest-rate decline in the investment-output ratio.

Figure 7: **Global long-term real interest rate**



Sources: IMF, Concensus Forecasts, Bloomberg and staff calculations

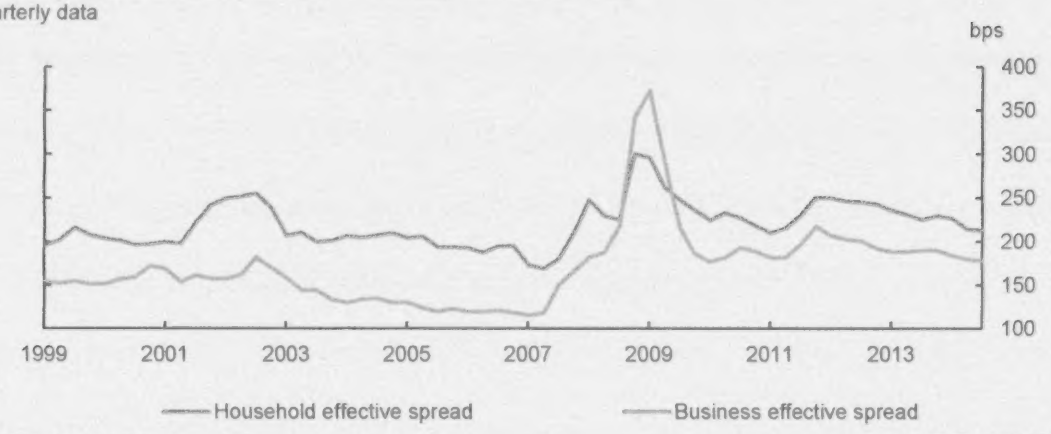
Last observation: July 2014

Many emerging-market economies (EMEs), especially in Asia, are pursuing policies that contribute to high savings rates. At the same time, elevated oil prices mean large current account surpluses in many countries that export oil. These factors caused the savings rate of EMEs to rise by about 10 percentage points in the decade before the crisis. Since 2008, EME savings have stabilized at an elevated level of 33 per cent of GDP. The IMF expects the EME savings rate to remain around this level through 2019. In addition, the savings rate of the advanced economies has been rising since 2008 and the IMF expects this trend to continue. This is consistent with the need for deleveraging, fiscal consolidation and balance-sheet repair in many advanced economies. Overall, the elevated level of global savings is putting downward pressure on the global neutral rate. This, in turn, is weighing on the Canadian neutral rate.

3.3 Higher credit spreads reduce investment and raise savings

The preceding analysis of savings and investment assumed that there is a single risk-free interest rate in the economy. In reality, households and firms do not generally have access to credit at the risk-free rate. When borrowing to finance investment or consumption, private economic agents pay a rate of interest equal to the risk-free rate plus a risk premium or credit spread. Since credit spreads influence private sector behaviour, they affect the level of the neutral rate.

Figure 8: **Effective spreads faced by households and businesses**
Quarterly data



Source: Bank of Canada
Last observation: 2014Q2

Since what matters for investment is the cost of capital, higher spreads would, all else equal, reduce investment. Similarly, higher household spreads would raise savings by reducing borrowing to finance consumption at a given risk-free rate. In a closed economy, weaker investment and stronger savings would directly translate into a lower interest rate. In an open economy, any reduction in borrowing that raised savings relative to investment would tend to weaken the external current account; over time, this would improve the NFA position and therefore reduce the country-specific risk premium. Thus, higher spreads tend to lower the neutral rate.

Relative to the pre-crisis benchmark, changes in spreads are not uniform across economies. In particular, spreads have not increased universally. However, increased costs of financial intermediation, partly as a result of needed financial regulatory reform, may cause spreads to settle at higher levels than in the pre-crisis period. A joint analysis by the Financial Stability Board and the Basel Committee on Banking Supervision attempted to quantify this effect. It found that a 3-percentage-point increase in banks' common equity Tier 1 capital ratio would raise lending spreads by around 45 to 50 basis points (BCBS and FSB 2010). The Bank's estimates for Canada are of a similar magnitude (Bank of Canada 2010). This could be reinforced by changes in investors' perceptions and pricing of risk in light of the experience of the crisis. As Caballero and Farhi (2014) show, an increase in the relative demand for safe assets can increase the spread between risky and risk-free assets and lead to a lower neutral rate.

Figure 8 shows effective spreads for households and firms in Canada. The 2013 average level of effective spreads was about 20 bps higher for households and 45 bps higher for firms, relative to the 1999–2006 average. Were this increase in spreads to persist, it would imply a lower neutral rate.

4 Estimates of the Neutral Rate in Canada

In the preceding section, we identified three longer-term forces influencing the Canadian neutral rate: (i) lower potential output growth, (ii) higher global savings, and (iii) higher credit spreads. In this section, we attempt to quantify the impact of these factors on the Canadian neutral rate. Our models take explicit account of domestic factors, while capturing foreign factors through a foreign interest rate variable. In order to mitigate the risks associated with any given model, we make use of multiple models. In all of the models, we derive a long-run, or steady-state, relationship between the domestic risk-free real interest rate and its key determinants. We then use estimates of the trend or low-frequency component of these determinants to estimate the neutral rate. Table 2 reports results from four approaches:

(i) pure interest parity
(ii) neoclassical growth model
(iii) reduced-form model
(iv) overlapping-generations model

The remainder of this section discusses each of these approaches in greater detail. Bank staff estimate that the neutral policy rate in Canada is between 1 and 2 per cent in real terms, or between 3 and 4 per cent in nominal terms. This range reflects the range of estimates from the various approaches summarized in Table 2.

Table 2: Summary of neutral rate estimates for Canada

Approach	Real neutral rate (%)	Nominal neutral rate (%)
Pure interest parity	1.00 to 2.00	3.00 to 4.00
Neoclassical growth model	1.75 to 2.00	3.75 to 4.00
Reduced-form model	1.00 to 1.50	3.00 to 3.50
Overlapping-generations model	1.50 to 2.00	3.50 to 4.00

4.1 Pure interest parity

Under the assumption of pure interest parity, international interest rate differentials can arise only as compensation for expected changes in exchange rates. In particular, there is assumed to be no risk premium. In the long run, expected changes in exchange rates should be zero, so interest rates should be fully equalized across countries.

This is equivalent to the Mundellian case discussed earlier. The neutral rate is equal to the foreign interest rate in the long run; domestic factors do not play a role. We assume that the relevant foreign neutral rate lies in the range of 1 to 2 per cent in real terms. Under the pure interest parity approach, the Canadian neutral rate would lie in the same range.

This range is bracketed by market rates that can be a proxy for longer-term expectations of interest rates. The GDP-weighted 10-year government bond real yield shown in Figure 7 lies just below the 1 per cent lower bound of our assumed range. On the other hand, the real five-year forward five-year rate in the U.S. is just above the 2 per cent upper bound of our range. However, it is important to note that inferences based on market prices may be distorted at present because current monetary policies in several major economies are designed to influence longer-term rates.

Alternatively, we can interpret the foreign neutral rate as the U.S. federal funds rate. If we assume 2 per cent inflation in the long run, in line with the Federal Reserve's target, the nominal range is 3 to 4 per cent. This range is broadly consistent with expectations of the target federal funds rate in the longer run in the Fed's September 2014 Summary of Economic Projections. Fifteen of the 17 Fed policy-makers surveyed projected a longer-run federal funds rate in the 3.25 to 4 per cent range. The median Fed policy-maker projection was 3.75. More generally, other forecasters' estimates fall within our assumed range (Table 3).

Table 3: Estimates of the nominal interest rate in the United States over the medium- to longer-run

	Estimate (%)	Date of estimate
Federal Reserve	3.75 (median)	June 2014
Deutsche Bank	3.50	July 2014
Goldman Sachs	3.75	June 2014
IMF	Less than 4.00	April 2014
Macroeconomic Advisers	3.75	June 2014
TD Economics	3.25	September 2014
Median	**3.75**	

4.2 Neoclassical growth model

Discussions about the neutral rate often focus on the implications of the standard neoclassical growth model. In this model, households have infinite horizons. In steady state, they demand a real interest rate that is approximately equal to

$$r + \gamma = \rho + \sigma g_c + n, \tag{3}$$

where r is the risk-free real rate, γ is the effective household credit spread, ρ is the household's subjective rate of time preference, σ is the inverse elasticity of intertemporal substitution, g_c is the steady-state growth rate of per capita consumption and n is the growth rate of the population:

- The rate of time preference measures impatience, so households demand greater compensation to postpone consumption when ρ is higher.

- Higher productivity growth implies higher per capita income in the future. Since households would like to smooth per capita consumption over time, a higher growth rate of per capita consumption creates an incentive for households to borrow (infinitely) in order to bring consumption forward. A higher real interest rate is required just to offset this incentive.
- The population growth rate affects the real interest rate in this model because households are assumed to be families that grow over time. Growth in the size of the family erodes the per capita value of the family's assets. Thus, with more rapid population growth, households require greater compensation for saving.[7]

Since per capita consumption and per capita income must grow at the same rate in steady state, we use per capita potential growth as a measure of long-run consumption growth. We set σ to 1.14 as in Dorich et al. (2013). Ideally, the discount rate, ρ, would be chosen to ensure that the actual *ex post* real effective interest rate ($r + \gamma$) was, on average, equal to the model-implied neutral rate over the period 1982–2013 (the period for which we have estimates of potential growth). However, the effective spread data are only available back to 1999. To work around this constraint, we choose $\rho - \gamma$ to make equation (3) hold on average. Using average projected per capita potential growth and population growth over 2014–16, this approach yields a real neutral policy rate of 2 per cent (4 per cent in nominal terms).

If spreads were to remain, say, 25 bps above the long-term average, this would reduce the neutral policy rate implied by equation (3) by an equal amount.[8] Thus, taking account of spreads, the neutral rate implied by the neoclassical model is 1.75 per cent in real terms or 3.75 per cent in nominal terms.

While this model is useful for identifying the basic determinants of the neutral rate, it has several drawbacks:

- It implies that the neutral rate is solely a function of the consumption-savings decisions of households. Investment demand plays no role in the determination of interest rates.
- In essence, the long-run savings supply curve is perfectly elastic at the real interest rate given by equation (3). If the long-run interest rate is greater than that implied by (3), then households will wish to save virtually all of their income; if the interest rate is less than that implied by (3), they will wish to borrow infinitely and consume as much as possible immediately. This knife-edge behaviour is unrealistic.
- It precludes any role for foreign determinants of the neutral rate.
- The rationale for why the population growth rate affects the neutral rate is not particularly compelling.

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[7] For a discussion of the inclusion of n in this expression, see Baker, DeLong and Krugman (2005).

[8] This assumes that all households face the spread. If some households, say those that do not borrow, faced the risk-free rate, then the increase in spreads would reduce the neutral rate by less than 25 bps.

- The fact that household members are infinitely lived makes it impossible to analyze the impact of life-cycle behaviour on aggregate savings.

The next two approaches attempt to rectify some of these deficiencies.

4.3 Reduced-form model

The preceding two approaches were extremes in the sense that they assumed that the neutral rate was determined solely by foreign factors or solely by domestic factors. In the reduced-form approach, we introduce an endogenous risk premium of the type discussed in section 2.2. This allows for the possibility that both domestic and global factors influence the Canadian neutral rate.

The basic ingredients of this approach are

(i) the balance-of-payments identity:

$$S_t - I_t = NX_t + r_t^{world} NFA_t, \tag{4}$$

(ii) the NFA accumulation equation:

$$NFA_t = \left(1 + r^{world}\right) NFA_{t-1} + NX_t, \tag{5}$$

(iii) a linear approximation to the interest parity condition:

$$r_t = r_t^{world} + E_t \Delta q_{t+1} + (\phi_0 - \phi_1 nfa_t), \tag{6}$$

where S is national savings, I is investment, NX is net exports, r_t^{world} is the interest rate on foreign assets, NFA is the net foreign asset position, r is the domestic interest rate and q is the exchange rate. The term in parentheses is the risk premium, which depends on the NFA-to-GDP ratio, $nfa = NFA/Y$.

We further assume that, in the long run, the savings rate ($s = S/Y$) and the investment-output ratio ($i = I/Y$) are given by linear behavioural relationships of the form:

$$s = \alpha_s + \beta_{s,r} r \tag{7}$$

$$i = \alpha_i + \beta_{s,r} r + \beta_{i,g} g, \tag{8}$$

where, as before, g is the growth rate of potential.

In Appendix A, we show that one can solve the steady state of this system for a reduced-form relationship of the form:

$$r = \alpha + \beta_0 g + \beta_1 r^{world}, \tag{9}$$

Table 4 reports estimates of this equation. In the first specification, we use the Bank of Canada's target for the overnight rate as the measure of r and the target federal funds rate as the measure of r^{world}. In the second specification, we use 10-year rates in Canada and the United States to measure interest

rates. In order to put the regression in policy rate space, we subtract the 1995–2014 average of the term spread from the 10-year yields.[9]

Both specifications suggest that foreign developments are more important than domestic potential growth in determining the Canadian neutral rate. If we take the midpoint of the 3 to 4 per cent range for the foreign neutral rate, together with 1.9 per cent for domestic potential growth (and rounding to the nearest 25 bps), both of these specifications imply a nominal Canadian neutral rate of 3.25 per cent. Allowing for a persistent increase in spreads could reduce this estimate by a further 25 bps.

One drawback of this approach is that it can be sensitive to the sample period used for estimation. For example, if we exclude the period since the global financial crisis, the Canadian potential growth rate is not significant in either specification. Using the point estimates based on the shortened sample period implies a neutral rate of about 3.5 per cent. In addition, the reduced-form approach is somewhat opaque since the underlying structural parameters are not uniquely identified by estimating the reduced form in equation (9). For instance, we would like to be able to evaluate the plausibility of the elasticity of the risk premium with respect to NFA implied by the estimates. But this is not possible because we cannot recover ϕ_1 from the reduced-form estimates.

Table 4: Regression results for reduced-form model

	Dependent Variable: Target overnight rate	Dependent Variable: 10-year Government of Canada bond yield
Constant (α)	0.49 (0.33)	-0.56** (0.20)
Canadian potential growth rate (g)	0.27* (0.16)	0.36*** (0.13)
Target federal funds rate (r^{world})	0.65*** (0.05)	
10-year U.S. Treasury yield (r^{world})		0.93*** (0.07)
R^2	0.83	0.91

Notes: The sample period is 1995Q1–2014Q2. Standard errors are in parentheses. Coefficient estimates are significant at 1% (***), 5% (**) and 10% (*) levels.

[9] Since inflation was stationary in both countries over this period, we simply use nominal interest rates in the regressions. Expected inflation affects the constant term in the regressions.

4.4 Overlapping-generations model

Using a structural overlapping-generations (OLG) model avoids the opacity of the reduced-form approach. It also overcomes several drawbacks associated with the standard neoclassical model:

- It allows for both savings and investment behaviour to influence the neutral rate (as opposed to just savings in the standard neoclassical model).
- It permits a role for both foreign and domestic factors (in contrast to just domestic factors in the standard neoclassical model).
- It provides a more plausible rationale for the role of population growth in the determination of the neutral rate. In the OLG model, as in section 3.1, population growth affects the neutral rate partly through its impact on investment demand.
- The overlapping-generations structure makes it possible to analyze the impact of life-cycle behaviour on aggregate savings.

For these reasons, Dorich, Mavalwalla and Mendes (2014) (DMM) develop an overlapping-generations (OLG) model of the Canadian economy. The DMM model is an open-economy model with an endogenous country-specific risk premium. Thus, it allows for both domestic and foreign factors to affect the Canadian neutral rate. In the model, there is a spread between the rate of return on capital and the risk-free rate. This spread can be interpreted as a firm credit spread. In contrast, households do not face a spread. Under a plausible calibration of the model, it yields a steady-state expression for the change in the interest rate of the form:

$$\Delta r = 0.5\Delta tlp + 0.4\Delta tli - 0.3\Delta fcs + 0.5\Delta r^{world}, \tag{10}$$

where Δ is the difference operator, tlp is trend labour productivity, tli is trend labour input, fcs is the credit spread faced by domestic firms and the other variables have the same definitions as before. Drawing on the changes relative to the pre-crisis average (Table 1), suppose that $\Delta tlp = -0.1$, $\Delta tli = -0.8$, $\Delta fcs = 0.5$ and $\Delta r^{world} = -1.5$. Using these in (10) gives a decline in the neutral rate of about 130 bps. If we assume that the real neutral rate in the mid-2000s was 3 per cent, then this implies a current real neutral rate of 1.75 per cent (rounding to the nearest 25 bps).

The simple exercise above gives a sense of the implications of the OLG model. However, it misses the rich dynamics that the model is capable of generating. In particular, the model can generate time paths for the neutral rate based on productivity, demographic, credit spread and foreign interest rate trends. Such dynamic simulations also capture the impact of the changing age distribution of the population. The simulations reported in DMM suggest a range from 1.5 to 2 per cent for the neutral rate.

5 Conclusion

In many ways, the neutral rate is like potential output: they are both time-varying, unobservable medium- to longer-run anchors. As with potential, the true value of the neutral rate is subject to considerable uncertainty. For this reason, we have employed several different approaches to estimating the neutral rate. This gives us greater confidence in the 1 to 2 per cent range for the real neutral rate (3 to 4 per cent nominal).

Nevertheless, there remain significant challenges. For example, the relative importance of domestic and foreign factors is highly uncertain. Our estimates of the reduced-form model are sensitive to the sample period, while results from the OLG model can vary with the calibration. In addition, there is uncertainty about the extent to which cross-country estimates of the link between country-specific risk premia and NFA positions are informative about the Canadian economy. More fundamentally, assessing the global neutral rate is fraught with difficulty. While savings and investment can flow fairly freely across borders in most advanced economies, this is not true of all countries. Some major emerging-market economies impose significant controls on international capital flows. This complicates the task of evaluating the impact of global savings and investment behaviour on the global neutral rate.

Another challenge is reconciling the model-based estimates of neutral with market rates. For example, the five-year forward five-year rate in Canada stood at about 2.5 per cent in August 2014 (0.5 per cent in real terms). This rate should reflect expectations of short-term rates five to ten years in the future plus a term premium. One plausible explanation for the very low five-year forward rate is that the high level of global liquidity has caused the term premium to become negative. Bauer and Diez de los Rios (2012) propose a model that can be used to decompose five-year forward rates into an expectations component and a term premium component. The model suggests that expectations of interest rates five to ten years in the future stood at 2.6 per cent in August 2014. This is consistent with a slightly negative term premium, but it still suggests that expectations are below the 3 to 4 per cent estimated range for the nominal neutral rate in Canada. As with all of the approaches discussed in this paper, there is uncertainty about the model's ability to identify the unobservable variables of interest. It is possible that the term premium is more negative than suggested by the Bauer-Diez de los Rios model. But it is also possible that the market believes headwinds are likely to persist for a prolonged period.

These challenges suggest avenues for future work. Bank staff will continue to (i) refine the methods used to assess the relative importance of global and domestic influences on the Canadian neutral rate, (ii) assess the effects of global savings and investment on the global neutral rate, and (iii) attempt to reconcile implied market expectations with model-based estimates of the neutral rate. Economists have been studying neutral or natural rates of interest for more than a century now. Much progress has been made, but much remains to be done.

Appendix A: Derivation of the Reduced-Form Neutral Rate Equation

For simplicity, we assume $\phi_0 = 0$. Let lowercase s, i and nx denote ratios to output. The steady state of equations (4)-(6) is

$$s - i = nx + r^{world} \cdot nfa$$
$$g_{ss} \cdot nfa = nx + r^{world} \cdot nfa$$
$$r = r^{world} - \phi_1 nfa,$$

where g_{ss} is steady-state output growth (in practice, this is interpreted as a very long-term average of potential growth). Combining these equations to eliminate nx and nfa yields:

$$s - i = -\left(\frac{g_{ss}}{\phi_1}\right)(r - r^{world}),$$

where g is output growth. Defining $\theta \equiv -(g_{ss}/\phi_1)$ makes this:

$$s - i = \theta(r - r^{world}).$$

Using the linear behavioural relationships for s and i in the previous equation and solving for r yields:

$$r = -\left(\frac{\alpha_s - \alpha_i}{\beta_{s,r} - \beta_{i,r} - \theta}\right) + \left(\frac{\beta_{i,g}}{\beta_{s,r} - \beta_{i,r} - \theta}\right)g - \left(\frac{\theta}{\beta_{s,r} - \beta_{i,r} - \theta}\right)r^{world}.$$

Or, more simply:

$$r = \alpha + \beta_0 g + \beta_1 r^{world}.$$

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